A Proposal to the MIT-Bates PAC
Precise Determination of the Proton Charge Radius

Spokespersons: H. Gao, J.R. Calarco

Contact person: H. Gao
email: haiyan@mit.edu, phone: (617)258-0256, fax: (617)258-5440

and the BLAST collaboration
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Abstract

The proton charge and current radii are fundamental quantities in physics. Precise determination of the proton charge radius is extremely important to the understanding of the proton structure in terms of quark and gluon degrees of freedom of Quantum Chromodynamics. It is also essential for high-precision tests of Quantum Electrodynamics using the hydrogen Lamb shift measurements. We propose a new precision measurement of the proton charge radius using a laser-driven polarized hydrogen internal gas target and the BLAST detector in the South Hall Ring at the MIT-Bates Laboratory. This measurement will fully utilize the unique features of the polarized internal target, polarized electron beam in the storage ring, and the BLAST large acceptance spectrometer. This measurement is expected to provide the most precise information on the proton charge radius from electron scattering, which will have significant impact on tests of QCD and QED.
I. INTRODUCTION

Nucleons (protons and neutrons) are fundamental building blocks of nuclei. Knowledge of the internal structure of protons and neutrons in terms of quark and gluon degrees of freedom of Quantum Chromodynamics (QCD) is not only essential for testing QCD, it also provides a basis for understanding more complex, strongly interacting matter at the level of quarks and gluons. Recent results from lattice QCD calculations suggest that the proton root-mean-square (rms) charge radius can be calculated from first principles with an uncertainty of only a few percent, and this field is rapidly evolving due to both improvements in computer architecture and new algorithms. Thus, precise information on this fundamental quantity is essential in terms of testing the QCD prediction from the lattice calculation.

Accurate information about this fundamental quantity is also essential in conducting high-precision tests of Quantum Electrodynamics (QED) from hydrogen Lamb shift measurements. The standard Lamb shift measurement probes the 1057 MHz fine structure transition between the $2S_{1/2}$ and $2P_{1/2}$ states in hydrogen. The hydrogen Lamb shift can be calculated to high precision from QED using higher order corrections. The proton rms charge radius is an important input in calculating the hadronic contribution to the hydrogen Lamb shift.

The two most precise and widely cited determinations of the proton charge radius in the literature give $r_p = 0.805(11)\text{ fm}$ [1] and $r_p = 0.862(12)\text{ fm}$ [2], respectively, differing from each other by more than 7%. While the recent precision hydrogen Lamb shift measurements [4–8] are in better agreement with the QED predictions using the smaller value of the proton charge radius without the two-loop binding effects, they are consistent with the larger value of the proton charge radius when two-loop binding effects are included in the QED calculations. We show in the following section that this discrepancy is dependent upon the data analysis and that the data are in fact consistent albeit with larger error bars. The past analyses also rely on an assumption of the ratio of the electric and magnetic form factors of the proton which has just recently been shown to be incorrect by recent experiments at Jefferson Laboratory. [9] Before accurate comparisons between theory and experiment can be made, both in QCD and QED, a new, precise measurement of the proton charge radius is urgently needed.

II. EXISTING MEASUREMENTS

The elastic electron-nucleon scattering has proven to be a very useful tool for probing the structure of nucleons. The squared momentum transfer $Q^2$ carried by the exchanged virtual photon in elastic electron-proton (ep) scattering is defined in terms of the four-momenta of the incident and scattered electron; $k$ and $k'$, respectively; by $Q^2 = -(k - k')^2$. By varying $Q^2$, the spatial distributions of charge and current in the proton can be mapped and these charge and current distributions are related to the proton electric and magnetic Sachs form factors $G_E^p$ and $G_M^p$.

In the low $Q^2$ region, below the two-pion production threshold; i.e., $Q^2 < t_0; t_0 = (2m)^2 < 2\text{ fm}^{-2}$; the energy transfer in the scattering process is negligible and the form factors can be taken as the Fourier transforms of the charge and current radial distributions $\rho_{\text{ch}}(r)$ and $\rho_{\text{cur}}(r)$. 

2
The rms charge radius of the proton is related to the proton electric form factor as:

\[
\frac{<r^2>}{6} = -\frac{dG_E^p(Q^2)}{dQ^2}\bigg|_{Q^2=0},
\]

with the boundary condition \( G_E^p(Q^2 = 0) = 1 \). To adequately determine the rms radius of the proton, high precision data of the proton form factors in the \( Q^2 \rightarrow 0 \) region are needed.

The elastic electron-proton scattering can be described theoretically using the one-photon exchange approximation. In this model the cross section is written as:

\[
\frac{d\sigma}{d\Omega}(E_0, \theta) = \sigma_{NS}(A(Q^2) + B(Q^2) \tan^2\{\theta/2\}),
\]

where \( \sigma_{NS} \) is the differential cross section for the elastic scattering from a point-like and spinless particle at incident energy \( E_0 \) and scattering angle \( \theta \). The two structure functions \( A(Q^2) \) and \( B(Q^2) \) are related to \( G_E^p \) and \( G_M^p \) by:

\[
A(Q^2) = [G_E^p(Q^2)^2 + \tau G_M^p(Q^2)^2]/(1 + \tau),
\]

\[
B(Q^2) = 2\tau G_M^p(Q^2)^2,
\]

with \( \tau = Q^2/(4M_p^2) \), where \( M_p \) is the proton mass. In the case of very low momentum transfer and small angle scattering, the magnetic contribution to the scattering process is suppressed and a precise separation of the form factors using the conventional Rosenbluth method [10] is not possible. However, as we discuss in detail below, their interference provides an easily measurable asymmetry when using a beam of polarized electrons in concert with a polarized proton target.

The most recent determination of the proton charge radius is from the work by Simon \textit{et al.} [2] in which the absolute differential cross sections of ep elastic scattering were measured at \( Q^2 \) values from 0.14 fm\(^{-2}\) to 1.4 fm\(^{-2}\). To extract \( G_E^p \) from the cross section measurement, they assumed the relation \( G_E^p/\mu_p = G_E^p \). Their best fitted value, which also incorporated data from Orsay [11] and Saskatoon [12], gave the rms charge radius of the proton as \( <r^2>^{1/2} = 0.862 \pm 0.12 \) fm. In their analysis \( G_E^p \) was assumed to have a polynomial \( Q^6 \) dependence, so that \( G_E^p = a_0 + a_1 Q^2 + a_2 Q^4 \), neglecting terms higher than \( Q^4 \). The effect of keeping the \( Q^6 \) term in the fit on the fitted value of \( <r^2>^{1/2} \) was studied and found to be negligible. The other widely cited value for the proton rms charge radius is from Hand \textit{et al.} [1], which contains a compilation of data from different experiments. In their analysis, they also assumed that \( \mu_p G_E^p = G_M^p \) and fitted \( G_E^p \) to the form \( G_E^p = 1 - 1/6 <r^2> Q^2 + AQ^4 \), using data for \( Q^2 \) up to 3 fm\(^{-2}\) to determine the parameter \( A \). From this fit, they determined the rms charge radius of the proton to be \( <r^2>^{1/2} = 0.805 \pm 0.011 \) fm.

In order to explore these discrepancies further, we re-fitted the \( G_E^p \) data from the experiments of Simon \textit{et al.} and those compiled by Hand \textit{et al.}. We concentrated on the data at the lowest values of \( Q^2 \) (\( Q^2 < 2.0 \) fm\(^{-2}\)) in order to maintain a consistent range of data between these two analyses. Furthermore, the model-independent determination of the proton charge radius is especially sensitive to the data in the \( Q^2 \rightarrow 0 \) region, according to equation (1). In our analysis of the data from Hand \textit{et al.} [1], in which we restricted
the fit to include only those data with $Q^2 \leq 2.0 \text{ fm}^{-2}$, we obtained a proton charge radius, $< r^2 >^{1/2} = 0.868 \pm 0.105 \text{ fm}$. When fitting the data compiled by Hand et al. up to $3.0 \text{ fm}^{-2}$, we obtained $< r^2 >^{1/2} = 0.809 \pm 0.060 \text{ fm}$, essentially in agreement with Hand's published result of 0.805 fm. In fitting these data, the following functional form was used: 

$$G_E^p = a_0 + a_1 Q^2 + a_2 Q^4.$$ 

Fitting the data from Mainz by Simon et al. [2] in the same way, we obtained $< r^2 >^{1/2} = 0.878 \pm 0.024 \text{ fm}$, within errors of their published result of 0.862 fm. In addition to these two sets of data, we also reanalyzed the data from Höhler et al. [3] and we obtain $< r^2 >^{1/2} = 0.863 \pm 0.057 \text{ fm}$, again within the consistent assumption concerning the ratio of the electric and magnetic form factors. Thus, these three determinations of the proton charge radius are consistent with one another within the errors.

In summary, by restricting the data sets to the most sensitive $Q^2$ region, we were able to obtain consistent results from all three data sets at the expense of increased uncertainties in the extracted values of $< r^2 >^{1/2}$. The most precise experimental information on the proton rms charge radius is from the measurement by Simon et al. which gives a $\sim 3\%$ measurement of this fundamental quantity. Though this measurement is in agreement with the results obtained from the analyses described above of the data from Hand et al. and Höhler et al. within errors, the errors of $< r^2 >^{1/2}$ obtained from the other two compilations are rather large, 12% and 7% respectively.

Thus, while the recent hydrogen Lamb shift measurements are in better agreement with QED predictions based on the smaller proton rms charge radius of 0.805 fm without the two-loop binding effects, self consistency argues that the these corrections are a necessary ingredient in the QED calculations, thus supporting a larger value of the proton charge radius (0.862 fm). Furthermore, there are good grounds to believe that the theoretical value of the Lamb shift in hydrogen will soon have a split-kHz accuracy, not including the uncertainty in the proton charge radius. Thus, to make meaningful high precision tests both of QED using Lamb shift measurements from hydrogen and of the expected improvements in the QED predictions, one needs a significant improvement in the precision in the determination of the proton rms charge radius. This includes taking into account the correct $Q^2$ dependence of the form factor ratio $\mu_p G_E^p/G_M^p$.

Recently, Rosenfelder [13] has reanalyzed some of the existing low-momentum-transfer scattering data from different groups and laboratories with Coulomb and recoil corrections included. This was done by calculating at each scattering angle and scattering energy the corresponding correction obtained from a partial wave program which includes recoil approximately. While a value of $r_p = 0.880(15) \text{ fm}$ is obtained which seems to be in good agreement with the one extracted from Lamb shift measurements, a few comments are needed. The result obtained is from a rather selective data sample; for example, the data compiled by Hand et al. [1] were not included. Furthermore, the $\chi^2$ per datum is too low, less than 0.15 for the best fit. Lastly, when data from different experiments were combined, it is always an issue how to take different systematic errors into proper account. The single measurement proposed here aims at an error half of that from combining five different experiments [13]. Furthermore, we will accurately determine the ratio $\mu_p G_E^p/G_M^p$ as a function of $Q^2$, eliminating the need to constrain it to unity as $Q^2$ increases.

We propose a new technique employing a polarized hydrogen gas internal target and a large acceptance detector in an electron storage ring, which will improve the precision of this
fundamental quantity by a factor of three compared with the most precise measurement that exists.

III. POLARIZED ELECTRON-PROTON ELASTIC SCATTERING

Polarization degrees of freedom in electron scattering have proven to be very useful in extracting information about small amplitude processes by isolating terms sensitive to the interference of the small amplitude with a much larger amplitude. The ability to selectively isolate certain combinations of amplitudes that either polarized electron beams or polarized targets have historically provided when used in isolation, is significantly enhanced by using them together. Thus, polarized electron scattering from polarized targets can provide important information on the nucleon spin structure and the electromagnetic structure of the neutron and proton.

Following the formalism of Donnelly and Raskin [14], the unpolarized differential cross section for elastic electron-proton scattering can be written in a slightly different form from Eq. 2 as:

\[
\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} f_{\text{recoil}}^{-1} ((1 + \tau) v_L G_E^2 + 2\tau v_T G_M^2),
\]

where \( f_{\text{recoil}} \) is the recoil factor,

\[
f_{\text{recoil}} = \frac{E}{E'}
\]

\( \sigma_{\text{Mott}} \) is the Mott cross section, and \( v_T \) and \( v_L \) are kinematic factors defined as follows:

\[
v_L = \frac{Q^4}{|\mathbf{q}|^4}
\]

\[
v_T = \frac{1}{2} \frac{Q^2}{|\mathbf{q}|^2} + \tan^2 \frac{\theta}{2},
\]

where \( \mathbf{q} \) is the three-momentum transfer vector, \( \mathbf{q} = \mathbf{k} - \mathbf{k}' \), \( \mathbf{k} \) and \( \mathbf{k}' \) are the three-momenta of the incident and scattered electron, respectively.

For longitudinally polarized electrons scattered from a polarized proton target, the differential cross section can be written [14] as:

\[
\frac{d\sigma}{d\Omega} = \Sigma - h \Delta,
\]

where \( \Sigma \) is the unpolarized differential cross section given by Eq. 5, and \( h \) is the electron helicity. \( \Delta \) is the spin-dependent differential cross section given by:

\[
\Delta = -\sigma_{\text{Mott}} f_{\text{recoil}}^{-1} [2\tau v_{T'} \cos \theta^* G_M^2 + 2\sqrt{2\tau(1 + \tau)} v_{T''} \sin \theta^* \cos \phi^* G_M G_E],
\]

where \( \theta^* \) and \( \phi^* \) are the polar and azimuthal proton spin angles defined with respect to the three-momentum transfer vector \( \mathbf{q} \) and the scattering plane (defined as the \( x, z \) plane with \( \hat{z} = \mathbf{q}/|\mathbf{q}| \) and \( \hat{y} = \mathbf{k} \times \mathbf{k}'/|\mathbf{k}||\mathbf{k}'| \)). \( v_{T'} \) and \( v_{T''} \) are kinematic factors defined as:
\[ v_{tT} = \tan \theta \frac{Q^2}{2 |q|^2 + \tan^2 \frac{\theta}{2}} \]  
(11)

\[ v_{TL} = -\frac{1}{\sqrt{2}} \frac{Q^2}{|q|^2} \tan \frac{\theta}{2} \cdot \]  
(12)

The spin-dependent asymmetry \( A \) is defined in terms of the polarized and unpolarized cross-sections as:

\[ A = \frac{\Delta}{\Sigma} = -\frac{2\tau v_{tT} \cos \theta^* G_M^p (1 + \tau) v_{TL}}{(1 + \tau) v_{L} G_E^p + 2\tau v_{T} G_M^p} \]  
(13)

The experimental asymmetry \( A_{\text{exp}} \) is related to the spin-dependent asymmetry of eqn. 13 by the relation

\[ A_{\text{exp}} = P_b P_t A \]  
(14)

where \( P_b \) and \( P_t \) are the beam and target polarizations, respectively. Flipping the electron helicity reverses the sign of the beam polarization and permits an experimental determination of \( A_{\text{exp}} \), and hence of \( A \).

A model-independent determination of the ratio of \( \frac{G_E^p}{G_M^p} \) can be precisely obtained by measuring the super ratio

\[ R = \frac{A_1}{A_2} = \frac{2\tau v_{tT} \cos \theta_1^* G_M^p}{2\tau v_{tT} \cos \theta_2^* G_M^p} \frac{2\tau (1 + \tau) v_{TL} \sin \theta_1^* \cos \phi_1^* G_M^p G_E^p}{2\tau (1 + \tau) v_{TL} \sin \theta_2^* \cos \phi_2^* G_M^p G_E^p} \]  
(15)

where \( A_1 \) and \( A_2 \) are elastic electron-proton scattering asymmetries measured at fixed \( Q^2 \) and at two different proton spin orientations corresponding to \((\theta_1^*, \phi_1^*)\) and \((\theta_2^*, \phi_2^*)\), respectively.

For a symmetric detector configuration as in the case of the BLAST detector, \( A_1 \) and \( A_2 \) can be measured simultaneously by forming two independent asymmetries w.r.t. the electron beam helicity in the left and right sectors of the detector, respectively. The acceptance of the two detector sectors are divided into symmetric bins which guarantees that the central values of the \( Q^2 \) for the \( A_1 \) and \( A_2 \) measurements are the same for each bin. The target spin angle is fixed by optimizing the combined figures-of-merit for this simultaneous asymmetry measurement described above. Since \( A_1 \) and \( A_2 \) will be measured simultaneously during the experiment, the super ratio \( R \) is independent of the determination of the beam and target polarizations. This is a unique advantage provided by the combination of a polarized internal gas target in an electron storage ring, and the symmetric large acceptance detector system. The overall error in the \( R \) measurement will be dominated by the statistical uncertainty of the measurement.

With the ratio \( \frac{G_E^p}{G_M^p} \) determined by the asymmetry measurements, only relative cross section measurements over the same \( Q^2 \rightarrow 0 \) region are needed in order to precisely determine the proton rms charge radius: because it is determined by the slope of \( G_E^p \) in the \( Q^2 = 0 \) limit, absolute cross section measurements are not necessary. Thus, utilizing polarization degrees
of freedom, one can determine the proton charge radius to high precision and thereby obtain the data of the best quality on the proton rms charge radius from electron scattering. This information will be sufficiently precise to permit meaningful tests of QED from measurements of the hydrogen Lamb shift and more importantly, will provide reliable tests of Lattice QCD calculations. The precision data on the proton electric and magnetic form factor ratio will also provide benchmark test of calculations of proton form factors.

IV. THE PROPOSED MEASUREMENTS

We propose to use a 275 MeV longitudinally polarized electron beam in the South Hall Ring at Bates and a laser-driven polarized hydrogen internal gas target to make these measurements. The BLAST detector will be employed to detect the elastically scattered electrons. With the BLAST magnet operating at the negative polarity and its maximum field strength, electrons of scattering angles between 15° and 72° can be covered for an incident electron beam energy of 275 MeV. This corresponds to a $Q^2$ coverage of 0.13 to 2.26 fm$^{-2}$. Figure 1 shows the event display of electron-proton elastic scattering at electron scattering angles between 10° and 80° at an incident electron beam energy of 275 MeV and a BLAST field of $-B_{\text{max}}$, where $B_{\text{max}}$ is the nominal maximum BLAST field.

For the determination of the form factor ratio $r = \frac{G_E}{G_M}$ from the ep elastic asymmetry measurement, the target spin will be aligned at an angle of $-160°$ with respect to the incident electron beam momentum direction. The holding field will be provided by a set of pan-cake coils whose orientation can be aligned to the desired target spin direction. To determine the proton charge radius, one needs to measure the slope of the proton electric form factor at $Q^2 = 0$. Thus, in addition to the form factor ratio measurements, relative differential cross section measurements are needed. The relative unpolarized differential cross section will be measured separately using an unpolarized electron beam and an unpolarized hydrogen target.

V. THE EXPERIMENT

A. Experimental Overview

This experiment will employ the polarized stored electron beam at 275 MeV, a polarized laser-driven hydrogen internal gas target and the BLAST detector. Each of these three major components of the experiment will be described below. In addition, unpolarized electron beams at 275 MeV and 300 MeV will be employed for the unpolarized relative differential cross section measurement. Since the experiment aims at a precision measurement of the proton rms charge radius, a section is devoted to address all important systematic uncertainties of the proposed measurement.

B. The polarized electron beam

A stored polarized electron beam with a polarization of 60% or higher at a beam energy of 275 MeV and an average beam current of 80 mA is required in this proposal for the
proposed measurement. Based on the experience with the strained GaAs crystals at Bates and NIKHEF, a peak current of 5 mA from the polarized source is an achievable goal once the flashlamp from NIKHEF is in use. One of the concerns about operating the South Hall Ring at such a low beam energy is the beam life time. Tests were done in 1994 at 330 MeV in which a beam 1/e life time of 4 minutes was achieved without any optimization. Based on the 1999 ring test with the N2 gas and the theoretical calculation, a beam life time in excess of 1000 seconds is realistic for a hydrogen target thickness of $2.6 \times 10^{18}$ atoms/sec. At 275 MeV, the horizontal damping time is 6.3 second. With a peak current of 5 mA from the polarized electron source, a beam life time of 1000 seconds and 15 stacks, an average beam current of 80 mA can be achieved. With the bulk GaAs crystal, an average beam current of 80 mA with a polarization of 35% is available today assuming the polarized electron beam can be stored in the ring. The proposed beam parameters are achievable in a year. Thus, the requested 80 mA of the average beam current at a polarization of 60% for this experiment is realistic. An early South Hall ring test at 275 MeV under the laser-driven target operating conditions will be very important to this experiment.

C. A Laser-driven polarized hydrogen gas target

The laser-driven polarized hydrogen target is based on the technique of spin-exchange optical pumping [15,16]. Optical pumping is a process of transferring angular momentum from a pump beam, typically a circularly polarized laser beam, to a sample of atoms. The polarized laser light optically pumps alkali atoms, which themselves become polarized along an axis defined by an external magnetic holding field.

Hydrogen (deuterium) atoms are polarized through spin-exchange collisions with the alkali atoms. At sufficiently large hydrogen densities, frequent H-H collisions enhance the probability of hyperfine interactions, which transfer polarization between the atomic electron and the nucleus, and the system approaches spin temperature equilibrium [17]. This happens even in a relatively strong magnetic field, which is required to overcome radiation trapping effect and achieve high efficiency for the optical pumping of alkali-metal atoms. In spin temperature equilibrium, the hydrogen nuclear and atomic polarizations are the same. Spin-exchange optical pumping of alkali-metal atoms in a hydrogen environment is an efficient way to polarize hydrogen atoms and atomic fluxes as high as $10^{18}$ atoms/second are typical with this type of source. Nuclear vector polarization as high as 50% for hydrogen can be achieved.

The performance of the laser-driven polarized H/D source using the spin-exchange optical pumping technique was first reported by Poelker et al in 1994 [18]. Their data showed the onset of spin temperature equilibrium in hydrogen and deuterium under various conditions, and nuclear vector polarizations for hydrogen and deuterium have been inferred from the measured electron polarization by assuming spin temperature equilibrium. Since that work, the existence of spin temperature equilibrium was established in a laser-driven polarized deuterium target [19] from the direct measurement of nuclear tensor polarization using a low energy fusion reaction and an indirect measurement [20] of the nuclear polarization in a laser-driven polarized hydrogen source was made with a Breit-Rabi Polarimeter. Currently, such a target is installed in the G region of the proton cooler ring at the Indiana University Cyclotron Facility (IUCF) where the target has performed well. The first nuclear physics
experiment [21] to study the spin-dependence of the deuteron wave function with this target started at IUCF in the spring of 1998.

The laser-driven polarized hydrogen target consists of a polarized source, a storage target cell, and an electron spin polarimeter for measuring the atomic polarization and monitoring the atomic fraction of the hydrogen flux, which consists of a mixture of atomic and molecular species. The polarized hydrogen source described here uses potassium as the spin-exchange medium. The glassware for the laser-driven source is one-piece, consisting of an RF dissociation tube, a spin-exchange cell with a short transport tube which connects to the target cell, and a potassium side arm.

The gas-feed system consists of pure hydrogen and deuterium gas supplies, a gas flow controller, and various valves and gauges. The hydrogen or deuterium molecules are dissociated into atoms by RF discharge. The spin-exchange cell and the aluminum storage cell are coated with dri-film to minimize surface recombination and depolarization effects from wall collisions. The atomic polarization is analyzed by an electron spin polarimeter which consists of a sextupole magnet, pumps, and a quadrupole mass analyzer (QMA) located at the focusing point of the sextupole magnet. The gas flow signal from the QMA is chopped so that a lock-in technique can be used to obtain a good signal-to-noise ratio.

The laser system consists of a Ti:sapphire laser and an Argon-ion pump laser. The Ti:sapphire laser is a standing wave laser with its spectral structure optimized by the combination of a pair of thin and thick intra-cavity uncoated etalons. The laser consists of 4–5 longitudinal modes with ~200 MHz separation. The spectral structure of the laser has reasonably good overlap with the Doppler profile of potassium atoms and optical pumping is efficient even in the absence of an electrical optical modulator (EOM). The typical magnetic holding field for efficient optical pumping is 1 kG, which is provided by a set of magnetic coils. A magnetic field of a few hundred gauss is necessary in the target cell region. To achieve high efficiency of optical pumping and to obtain large polarization in the target, two identical sets of the laser system described above will be simultaneously employed. This is also the laser system currently used in the IUCF laser-driven target program.

For a laser-driven polarized hydrogen target, the following was achieved at IUCF during the data taking of the experiment CE66 in Dec. 1997: an effective target nuclear polarization of 0.25 at a hydrogen gas flow rate of ~ \( 1.0 \times 10^{18} \) atoms/second. In this proposal, we propose to run this experiment at a hydrogen flow rate of \( 2.0 \times 10^{18} \) atoms/second with an effective polarization of 0.3. We are optimistic that we can achieve this performance in a realistic time scale for this experiment to run. Currently, such a target is being built at MIT and we aim at achieving the above mentioned goal by summer of 2001, which coincides well with the current schedule of the BLAST construction and commissioning.

For the nominal BLAST storage cell dimensions, a hydrogen flow rate of \( 2.0 \times 10^{18} \) atoms/second and an average beam current of 100 mA correspond to a luminosity of \( 1.3 \times 10^{33} \) (atoms cm\(^{-2}\) s\(^{-1}\)) under the normal laser-driven target operating conditions. The luminosity for the polarized \(^3\)He target is \( 1.0 \times 10^{33} \) (atoms cm\(^{-2}\) s\(^{-1}\)) in the BLAST TDR. Whether the proposed hydrogen target thickness is an issue for the beam lifetime can only be addressed by a ring test.

An alternative to the laser-driven target is the Atomic Beam Source (ABS), which is a well-established technology. Such a target is currently being installed at the BLAST internal target region. A target polarization of 70% at a flow rate of \( 7 \times 10^{16} \) can be achieved. The
proposed figure-of-merit of the laser-driven target is 2.5 times of that of the atomic beam source. If the proposed target thickness from the laser-driven source proves to be a serious issue for the beam life time, running this experiment with the ABS is a possibility with the following scenario for the stored beam. With an injection current of 15 mA from the polarized electron source (corresponding to a linac peak current of 5 mA), a beam life time of 10 minutes (with the ABS target thickness, the beam life time is expected to be much longer than 10 minutes) and an injection period of 6 seconds, the theoretical maximum stored current is 1000 mA. With the theoretical maximum stored current of 1000 mA and taking 300 mA as a limit on the maximum stored current, one can achieve an average beam current of 143 mA under the following optimized conditions: $T_{\text{ful}} = 3.5$ minutes and $T_{\text{data}} = 7.4$ minutes. With 40% additional polarized beam time than what is requested here, the same physics goal can be accomplished.

D. Impact of the LDS holding field

As discussed earlier, we will form the super ratio $R$ by simultaneously measuring two independent asymmetries $A_1$ and $A_2$ to determine the proton form factor ratio $f = \frac{\sigma_p}{\sigma_e}$. The target spin will be oriented at $-160^\circ$ with respect to the electron momentum direction. This holding field will contribute a small component which is transverse to the beam direction. In order to compensate for this, we plan to install steering dipoles in a configuration that is similar to what was done at NIKHEF: one dipole was installed just upstream and one just downstream of the target area. With each dipole contributing half the deflection of the transverse component of the holding field (and in the opposite direction), the beam is slightly (about 2.5 mm) off the nominal beam axis inside the cell but, on average, parallel to it. An alternative scheme is to allow the beam into the cell without deflection and to fully compensate for the deflection downstream. This requires more steering dipoles, 6 to be exact, but there is sufficient space along the downstream beamline to do this. In this case the beam enters the target cell on axis and parallel to it, but is off axis by $\sim 0.4$ mm at the exit and at an angle of $\sim 20^\circ$.

E. The BLAST detector

We propose to run this experiment at an electron beam energy of 275 MeV and 300 MeV. The detection of the elastically scattered electrons between scattering angles of 15° and 7° is required so as to cover a $Q^2$ range between 0.13 and 2.26 fm$^{-2}$. This requires operating the BLAST magnet at the opposite polarity and at its nominal maximum field strength. Both sectors of the BLAST detector will be employed to detect the elastically scattered electrons. All detectors will be operated in their standard configurations.

To calculate the angular coverage of the planar drift chambers, a simple Monte Carlo simulation code was used to generate elastic events and to track them through the drift chambers. Events were generated to originate uniformly within the target cylinder ($r=6.25$ mm, $l=400$ mm) and to have an isotropic distribution for the electron scattering angles ($10^\circ \leq \theta \leq 90^\circ$, $-15^\circ \leq \phi \leq 15^\circ$). The momentum for the electron was calculated from the
polar scattering angle $\theta$ assuming an incident kinetic energy of 275 MeV and elastic scattering. The electron was then tracked through the BLAST magnetic field. The corresponding recoil proton scattering angles and momenta were calculated from the kinematics and similarly tracked through the drift chamber. Figure 1 shows the distribution of electron momenta versus polar scattering angle for all events generated. Figure 2 is a similar distribution but with the requirement that the electron be detected in all three planar drift chambers. This shows that the drift chambers detect electrons with scattering angles between 10° and 80°.

F. Beam time estimate

In our estimate of the running time for the form factor ratio measurement, we chose a scattering angle bin of $\Delta \theta = 48$ mrad and assumed the total azimuthal angular acceptance ($\pm 15^\circ$) of the BLAST detector. For the target and beam parameters, we used the following realistic numbers: a hydrogen flow rate of $2 \times 10^{18}$ atoms/sec, an effective target polarization of 30% and a standard BLAST storage target cell, an average electron beam current of 80 mA and a beam polarization of 60%. Table I lists the statistical uncertainties for the super ratio $R$ measurement as a function of $Q^2$. These errors are estimated corresponding to a polarized beam time of 800 hours.

To determine the proton charge radius, one needs to measure the slope of the proton electric form factor at $Q^2 = 0$. Thus, relative differential cross section measurements together with the form factor ratio measurement in the $Q^2 \to 0$ region will be sufficient to determine the slope of $G_E^p$ at $Q^2 \to 0$. In order to determine the relative differential cross section to the required accuracy, a scattered electron energy bin of 3 MeV ($\pm 1.5$ MeV) is necessary from the consideration of the finite resolutions in the electron momentum and scattering angle determinations. This 3 MeV energy bin was chosen based on the Monte Carlo simulation results (see Section H on the discussion of the systematic uncertainties). This corresponds to a scattering angle bin of $\sim 70.0$ mrad, and the total BLAST out-of-plane angular acceptance is used in estimating the beam time. With this set of detector parameters, we will obtain a total of 30 $Q^2$ bins for the relative differential cross section measurement in the $Q^2$ range of interest, taking into account both sectors of the BLAST detector. The relative differential cross section measurement will be repeated twice at an incident electron beam energy of 275 MeV which allows binning the data in different ways. Thus, we will have three independent sets of measurements of the relative differential cross section. The proton rms charge radius will be extracted by combining each set of the differential cross section data and the proton form factor ratio data. The proton rms charge radius will be determined to a precision of 0.007 fm by combining three independent extracted $r_p$ values.

The differential cross section measurement requires unpolarized electron beam and target, and the beam time required is 50 hours. This corresponds to a statistical uncertainty of better than 0.1% for all $Q^2$ bins for all three sets of the measurements at 275 MeV incident electron beam energy. In addition, we request 50 hours of beam time at an incident electron beam energy of 300 MeV for a separate three sets of relative differential cross section measurement corresponding to a statistical uncertainty of better than 0.1% for all $Q^2$ bins in a $Q^2$ range between 0.15 to 2.65 fm$^{-2}$ for each data set. This additional relative cross section measurement will help to cross check our understanding of the systematic uncertainties in extracting the proton charge radius.
We request a beam time of 900 hours (37.5 days) for the production run which includes 800 hours of polarized beam and 100 hours of unpolarized beam. In addition, we request 100 hours for detector checkout and calibration. Thus, in total we request 1000 hours (42 days) of beam time for this experiment.

G. Uncertainties

To determine the proton electric form factor from the relative differential cross section measurement, one needs to use the proton form factor ratio determined from the asymmetry measurement to substitute for the magnetic part of the contribution to the differential cross section so as to extract the electric form factor of the proton. Thus, the uncertainty in determining the electric form factor will be dominated by how well one can determine the magnetic fraction of the contribution to the differential cross section and how well the relative differential cross section can be measured.

Three sources of error contribute to the total error in the determination of the proton form factor ratio: the statistical error in the measurement of the super ratio $R$, the uncertainty in determining the target spin angle $\theta_1^*$ and $\theta_2^*$ in the two detector sectors, respectively. Since $\theta^*$ is defined with respect to the three-momentum transfer vector $q$, both the uncertainty in determining the direction of $q$ vector and the uncertainty in aligning the target spin contribute to the determination of $\theta^*$. This systematic uncertainty has been studied carefully. The target spin can be aligned to an accuracy of 0.06° by carefully mapping the magnetic field in the target region assuming that one can measure the magnetic field to a realistic accuracy of 0.5 gauss. The uncertainty in determining the $q$ vector direction has been calculated for each bin based on the spectrometer momentum and angle resolutions obtained from the GEANT simulation. Table II lists the error contribution from each of the three sources. The overall error is dominated by the statistical uncertainty in measuring the super ratio $R$. Fig. 3 shows the projected overall errors for the $G_E/G_M$ ratio measurement.

At the $Q^2$ range of this experiment, the error from the proton form factor ratio determination will contribute an uncertainty of 0.17% (relative error) to the electric form factor at $Q^2 = 2.2$ fm$^{-2}$ and an uncertainty of 0.077% at $Q^2 = 0.13$ fm$^{-2}$ because the magnetic contribution is significantly suppressed at forward angles.

As mentioned in the last section, the projected statistical uncertainty in the relative differential cross section measurement is better than 0.1% for all $Q^2$ bins. Thus, it is very important to address all systematic uncertainties associated with the relative differential cross section measurement, especially for a large acceptance spectrometer with moderate resolutions. There are two important issues in determining the relative differential cross section accurately, which are an artifact of binning the data in small bins of scattered electron energy $E'$ and scattering angle $\theta$. One is determining the mean value of $Q^2$ for a given bin in $E'$ and $\theta$, and the other is determining what fraction of the events belonging to a given bin spill over into neighboring bins due to the finite momentum and angular resolution of the detectors. The mean value of $Q^2$ for a given bin can be determined if one knows all the materials the electron sees on its way to the detector and one can compute the most probable energy loss which takes place due to these intervening materials. Determining the fraction of events leaving and entering a given bin however is more difficult and one needs to use a Monte Carlo simulation to determine a correction factor for each bin. Such
a simulation was done and the correction factors as well as the uncertainty in determining the correction factors were calculated. The procedure is described below.

Elastic e-p scattering events were generated according to the elastic electron-proton scattering cross section between scattering angles corresponding to \( \theta \) 15-72 degrees, for a beam energy of 275 MeV (Since the elastic kinematics is over determined one needs to generate just the angle randomly while the scattered electron energy is given by the kinematics). These events were then smeared by a Gaussian of width equal to the nominal resolution of the detectors. We used the numbers \( \sigma_{\theta} = (0.4 + 0.1E) \% \) and \( \sigma_{\theta} = 1 \text{ mr} \). In addition we made the assumption that we would be able to reconstruct the tracks exactly and account for all energy loss in the intervening materials. Next by binning the generated and smeared events in small bins of \( E' \) and \( \theta \) we can calculate the correction factor for each bin as the ratio of the generated to the smeared events. Since the momentum resolution was responsible for the bulk of the smearing the size of the bin in energy was varied in order to obtain an optimum bin size where the correction would be on the average about 20 %. The optimum bin size was found to be \( \pm 1.5 \text{ MeV} \) in \( E' \). However, the effect due to this correction factor cancels out in the asymmetry measurement. Thus, one can work with a smaller \( E' \) bin in the super ratio \( R \) measurement. Next the resolutions themselves were varied for the optimal bin size to determine the uncertainty in the correction factors. It was found that if the resolutions are known with an uncertainty of \( \pm 2\% \), the correction factors would have an uncertainty of \( \pm 1\% \). And there is little variation in this uncertainty over the whole energy range of interest. Since the experiment involves measuring the relative change between bins the uncertainty introduced in the measurement will be smaller than the absolute uncertainty in the correction factor. We expect the overall uncertainty in the relative differential cross section measurement to be 0.5\%. Table III lists the sources of error to the determination of \( G_E \) (relative) and the total error which is dominated by the overall error in determining the relative differential cross section. The binning for the relative differential cross section measurement is different from that of the proton form factor ratio measurement as was discussed previously. In table III, we list the errors using the binning for the form factor ratio measurement to symbolically represent the error due to the relative cross section measurement.

Although the experiment involves making a relative cross section measurement, one needs to know the exact variation in the acceptance from one bin to the other. In addition, since we will be relying on the reconstruction to define the bins, one has to calibrate the reconstruction. Variation in the acceptance can be caused by variation of the detector efficiencies and hence the detector efficiencies have to be monitored continuously during the run. This means that the efficiencies of the wire chambers have to be studied in great detail and the wire chamber acceptance has to be sub-divided into small regions and the efficiency has to be determined in each of these small segments. This calibration procedure can be carried out with a portable 6 MeV small electron linear accelerator or a UV laser. With a proper calibration the counts in each segment can be monitored continuously during the experiment from run to run and would give us the variation in the detector efficiency with time.

The issue of variation of the relative acceptance from bin to bin, where the bins are defined by software cuts on the reconstruction, can be addressed by at least two different methods. First, utilizing the symmetry of the two sectors, an acceptance defining collimator slit and a target collimator will be used to define a small region of the acceptance in one sector while
the corresponding region in the other sector is reconstructed. Next the slit together with the target collimator will be installed in the other sector. Thus the hardware definition of the acceptance and the software reconstruction can be swapped and the experiment repeated. This kind of calibration procedure will be repeated to cover different parts of acceptance so that a reasonable calibration of the reconstruction across the acceptance can be achieved and thereby determine the variation of the acceptance from bin to bin.

Alternatively, the silicon strip recoil (SSR) detectors can be used in coincidence with the scintillators in the opposite sector. The SSR detectors provide segmentation in both $x$ and $y$ or, correspondingly, $\theta_p$ and $\varphi_p$. In conjunction with the reconstruction of the event vertex to an accuracy of $3 \text{ mm}$ along the target axis and $\leq 1 \text{ mm}$ transverse to it, the SSR detectors map out the location of an “expected” electron, which should be observed as a scintillator Čerenkov coincidence. Any loss of efficiency can be continuously monitored. In addition, because of the linear extent of the target, any given pixel in the SSR detectors maps onto several adjacent scintillators, and vice versa, each scintillator can be seen in coincidence with adjacent SSR pixels providing a very nice redundancy.

The form-factor analysis of electron-proton scattering is based on one-photon exchange diagram. The validity of it is based on the assumptions that higher-order corrections are negligible. However, for any precision measurement as what is proposed here, it is important to address the higher-order electromagnetic corrections. Drell and Fubini [23] calculated the higher-order corrections and showed that the form factor analysis of electron-proton scattering is accurate to $\sim \frac{1}{\alpha^3}$ for all angles and energies into the GeV range. They have used dispersion theory methods to formulate the electron-proton scattering amplitude in a manner which allows them to evaluate the $e^4$ contribution due to Compton scattering of the virtual intermediate photons by the proton.

Greenhut [24] calculated the two-photon exchange contribution to electron-proton scattering in second Born approximation using potentials representing the charge and magnetic moment distributions of the proton. The resonance contribution was calculated using a fit of the proton Compton scattering data. The numerical results showed that up to order $\alpha^3$, the two-photon exchange contribution is less than 0.8% at the kinematics of the proposed experiment in the case of the unexcited intermediate proton states and the resonance contribution at the kinematics of this experiment is negligible. Furthermore, one can follow the formalism derived by Greenhut to apply this correction to each $Q^2$ bin and the relative error from bin to bin due to this correction is negligible.

Recently, Rosenfelder [13] applied the Coulomb correction to elastic electron-proton scattering and showed that the analysis of the electron scattering data including Coulomb correction lowers the $\chi^2$ of the fit and increase the proton radius by about $(0.008 - 0.013)$ fin depending on the fit strategy. The Coulomb correction is calculated using a standard partial wave program which solves the Dirac equation in the electrostatic potential of the proton. The coulomb corrections are found to be small but positive, ranging from $0.4 - 0.9 \%$. These results are in good agreement with those from Greenhut, though in the latter case both the Coulomb and magnetic corrections have been taken into account using the second Born approximation.

To estimate the total error in determining the proton charge radius, the following procedure has been used. The dipole form factor parametrization was used to generate $G_E^p$ as a function of $Q^2$ using the same range and step size as those proposed, randomized by the
total error in determining the relative proton electric form factor. A polynomial fit was then performed to the second order of $Q^2$. The proton charge radius can be determined to a precision of 0.007 fm from this experiment. Fig. 4 shows the projected measurement together with the world data on $r_p$ from different experiments including the new analysis discussed in this proposal. Also shown is the most recent determination of $r_p$ from the hydrogen Lamb shift measurement using the state-of-the-art QED calculation which includes the three loops [25].

H. Collaboration background

This collaboration has extensive experience in running experiments with polarized external and internal gas targets. By the time this experiment runs with BLAST, many members of this collaboration should have adequate experience with running the BLAST detector. The MIT group is responsible for providing the laser-driven polarized hydrogen gas target for this experiment. We request technical and mechanical support from the laboratory for the installation of the target at Bates. In addition, we request beam development time in establishing stable, optimal running conditions for this experiment at 275 and 300 MeV.

VI. ACKNOWLEDGEMENT

We thank T.W. Donnelly, J.L. Friar, X.D. Ji, D. Kleppner and M. Ramsey-Musolf for stimulating discussions. We acknowledge T. Wise and W. Haeberli from University of Wisconsin, Madison for their help in constructing the storage target cells for the laser-driven target.
TABLES

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<th>$E'$ (MeV)</th>
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TABLE I. The projected statistical uncertainties for the super ratio $R$ measurement, where $R = \frac{A_1}{A_2}$, as a function of $Q^2$ for a beam time of 800 hours.
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**TABLE II.** The relative error in the determination of the proton form factor ratio, $\frac{\Delta r}{r}$, where $r = \frac{G_E}{G_M}$. The contribution to the relative error in $r$ due to the statistical uncertainty of the super ratio $R$ measurement (shown in the second column) corresponds to a beam time of 800 hours. The third and forth columns are the relative errors in $r$ due to the target spin angle uncertainties for the two detector sectors, respectively. The total relative error is the quadrature sum of all three contributions.
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<td>0.297</td>
</tr>
</tbody>
</table>

**TABLE III.** The relative error in the determination of the proton electric form factor (relative). The total error is the quadrature sum of the contributions from the determination of the proton form factor ratio and the determination of the relative differential cross section.
FIG. 1. Monte Carlo simulation of the distribution of electron momenta versus polar scattering angle for all elastic events generated at an incident electron beam energy of 275 MeV.
FIG. 2. Same as figure 1 with the requirement that the electron be detected in all three planar drift chambers.
FIG. 3. The total uncertainties of the proton form factor ratio measurement.
FIG. 4. The proposed measurement of $r_p$ together with the existing data from various experiments and analyses.
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